

Special type of matrices :

- 13) Hermitian & skew Hermitian
- 14) Orthogonal matrices
- 15) Nilpotent matrices
- 16) Involuntary matrices
- 17) Idempotent matrices

Elementary operations (row) : (3)

- 1) Interchange $R_i \leftrightarrow R_j$
- 2) Multiplication by a constant : $R_i \rightarrow c R_i$
- 3) Multiplication by a constant & addition to another row
 $R_i \rightarrow R_j + c R_i$ or $R_i + c R_j$ or $c_1 R_i + c_2 R_j$

Echelon form : $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 5 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 1 & -1 & -1 & 3 & 2 \\ 0 & 1 & 5 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Eg. $\begin{bmatrix} 1 & -1 & -1 \\ x_1 & 0 & 5 \\ 0 & 0 & 1 \end{bmatrix}$

Eg 1) $\begin{bmatrix} 0 & 2 & 3 & 0 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & 5 \end{bmatrix}$ $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & -2 & 5 \\ 2 & 3 & 1 & 5 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

$$\begin{array}{rrrr} 2 & -2 & -4 & 10 \\ 2 & 3 & 1 & 5 \\ \hline 0 & -5 & -5 & 5 \end{array}$$

$R_2 \rightarrow 2R_1 - R_2$

$$\begin{bmatrix} 1 & -1 & -2 & 5 \\ 0 & -5 & -5 & 5 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

$R_3 \rightarrow 2R_2 + 5R_3$

$$\begin{bmatrix} 1 & -1 & -2 & 5 \\ 0 & -5 & -5 & 5 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

$$\begin{array}{rrr} -10 & -10 & 10 \\ 10 & 15 & 40 \\ \hline 0 & 5 & 50 \end{array}$$

$$R_3 \rightarrow 2R_2 + R_1$$

$$\left[\begin{array}{cccc|ccc} 1 & -1 & -2 & 5 & 0 & 5 & 50 \\ 0 & -5 & -5 & 5 & & & \\ 0 & 0 & 5 & 50 & & & \end{array} \right]$$

$$2) \left[\begin{array}{cccc} \textcircled{1} & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 5R_1$$

$$R_3 \rightarrow R_3 - 9R_1$$

$$R_4 \rightarrow R_4 - 13R_1$$

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & \textcircled{4} & 8 & 12 \\ 0 & 8 & 16 & 24 \\ 0 & 12 & 24 & 36 \end{array} \right]$$

$$R_3 \rightarrow R_3 - 2R_2$$

$$R_4 \rightarrow R_4 - 3R_2$$

$$\Rightarrow \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 4 & 8 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \left. \vphantom{\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 4 & 8 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}} \right\}$$

Note: Row echelon form need not be unique.

Row Reduced echelon form: 1) A should be in row echelon form.

2) All leading entries of row should be 1.

3) Corresponding to leading 1, all other entries in the column should be zero.

$$\left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 4 & 8 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{R_2 \rightarrow \frac{1}{4}R_2} \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$R_1 \rightarrow R_1 - 2R_2$$

$$\left[\begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array} \right] \left. \vphantom{\begin{array}{cccc} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}} \right\} \rightarrow \text{RREF.}$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{RREF.}$$

eg: H.W.

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 9 \\ 1 & 5 & 12 \end{bmatrix}$$

$$\stackrel{\text{Ans}}{=} \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & 7/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\text{Rank } A = 2$$

Note 1 RREF must be Unique.

REF:

$$1) \begin{bmatrix} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} \textcircled{1} & -1 & -2 & -5 \\ \underline{2} & 3 & 1 & 5 \\ 0 & 2 & 3 & 8 \end{bmatrix}$$

$$R_2 \rightarrow R_2 - 2R_1$$

$$= \begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & \textcircled{5} & 5 & 15 \\ 0 & 2 & 3 & 8 \end{bmatrix}$$

$$0 \rightarrow 2R_2 - 5R_3$$

$$R_3 \rightarrow 2R_2 - 5R_1$$

$$\begin{array}{cccc} 0 & 10 & 10 & 30 \\ 0 & 10 & 15 & 40 \\ \hline 0 & 0 & -5 & -10 \end{array} =$$

$$\left[\begin{array}{cccc|c} 0 & 2 & 3 & 8 & \\ \hline 1 & -1 & -2 & -5 & \\ 0 & 5 & 5 & 15 & \\ 0 & 0 & -5 & -10 & \end{array} \right]$$